

JEE QUESTION: MECHANICAL PROPERTIES OF FLUIDS

- 1) A liquid of density ρ flows steadily through a horizontal pipe whose radius varies with distance x as

$$r(x) = r_0(1 + \alpha x)$$

ANSWER

$$A(x)v(x) = \text{constant}$$

$$\pi r_0^2 v_0 = \pi r(x)^2 v(x)$$

$$v(x) = \frac{v_0 r_0^2}{r_0^2 (1 + \alpha x)^2}$$

BERNOULLI'S THEOREM

$$P + \frac{1}{2} \rho v^2 = \text{constant}$$

$$P_0 + \frac{1}{2} \rho v_0^2 = P(x) + \frac{1}{2} \rho v(x)^2$$

Substitute $v(x)$,

$$P(x) = P_0 + \frac{1}{2} \rho \left(v_0^2 - \frac{v_0^2}{(1 + \alpha x)^4} \right)$$

$$P(x) = P_0 + \frac{1}{2} \rho v_0^2 \left(1 - \frac{1}{(1 + \alpha x)^4} \right)$$

2. A large tank contains water upto height H . Two small holes are made at depths h_1 & h_2 from the water surfaces. For what condition on h_1 & h_2 will be horizontal range of water jets be equal?

ANSWER

Speed of efflux

$$v = \sqrt{2gh}$$

→ Time

height from hole to ground: $H - h$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Horizontal range

$$R = vt = \sqrt{2gh} \cdot \sqrt{\frac{2(H-h)}{g}}$$

$$R = 2\sqrt{h(H-h)}$$

EQUAL RANGES

$$h_1(H-h_1) = h_2(H-h_2)$$

$$\Rightarrow h_1 + h_2 = H$$

3) A capillary tube of radius r is inclined at an angle θ with the vertical. Find the length of the liquid column inside the tube.

ANSWER

Vertical height risen

$$h = \frac{2T \cos \theta}{\rho g r}$$

If the tube is inclined,

$$h = L \cos \theta$$

$$L = \frac{h}{\cos \theta}$$

$$\Rightarrow L = \frac{2T \cos \theta_c}{\frac{\rho g r}{\cos \theta}} = \frac{2T \cos \theta_c}{\rho g r \cos \theta}$$

Properties of Fluid

Q Water is stored in a tank of height H . A small hole is made at height h from the bottom. Find horizontal distance where water strikes the ground.

Sol: Velocity from hole:

Using Torricelli (from Bernoulli):

$$v = \sqrt{2g(H-h)}$$

Time of fall:

height from hole to ground = h

$$t = \sqrt{\frac{2h}{g}}$$

horizontal distance:

$$x = v \cdot t$$

$$= \sqrt{2g(H-h)} \times \sqrt{\frac{2h}{g}}$$

$$x = \sqrt{4h(H-h)} = 2\sqrt{h(H-h)}$$

Q Water rises in a capillary tube of radius r to height h . Show that surface energy gained equals gravitational potential energy.

Sol: Surface area of liquid column:

$$A = 2\pi r h$$

$$\text{Energy} = \gamma E_s = \gamma \cdot A = 2\pi r h \gamma$$

Gravitational potential energy:

$$\text{Mass: } m = \rho \pi r^2 h$$

height of COM: $\frac{h}{2}$

$$E_g = mg \frac{h}{2} = \rho \pi r^2 h g \cdot \frac{h}{2}$$

$$= \frac{1}{2} \rho \pi g r^2 h^2$$

Use capillary formula:

$$h = \frac{2T}{\rho g r}$$

substitute into E_g

$$E_g = \frac{1}{2} \rho g \pi r^2 \left(\frac{2T}{\rho g r} \right)^2$$

$$= \frac{1}{2} \pi r^2 \cdot \frac{4T^2}{\rho^2 g r^2}$$

$$= \frac{2\pi T^2}{\rho g}$$

simplify E_s

$$E_s = 2\pi r h T$$

$$\text{substitute } h: 2\pi r \cdot \frac{2T}{\rho g r} \cdot T$$

$$= \frac{4\pi T^2}{\rho g}$$

Compare: factor difference shows $\rightarrow E_s = 2E_g$

half goes to raising liquid, half to forming surface

Q Water rises to height h in a capillary tube.

If the radius of the tube is doubled, what happens to the height?

$$\text{sol: } h = \frac{2T}{\rho g r} \quad \text{so } h \propto \frac{1}{r}$$

$$\text{If, } r \rightarrow 2r$$

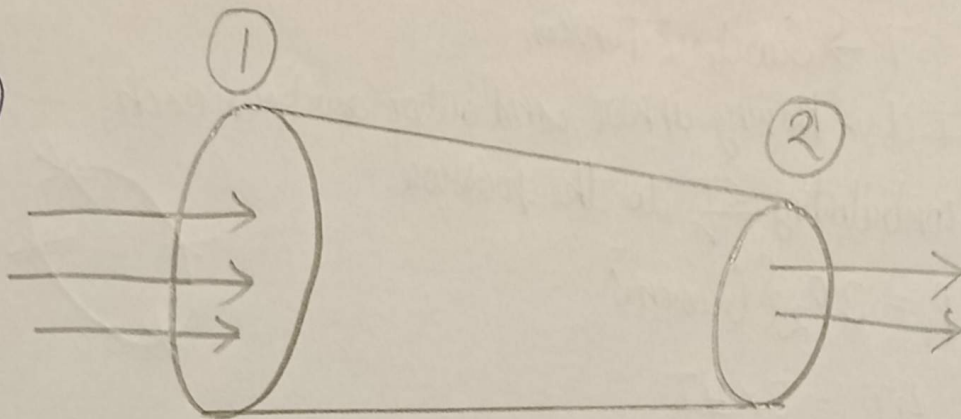
$$h' = \frac{h}{2}$$

$$\therefore h' = \frac{h}{2}$$

JEE Questions

①

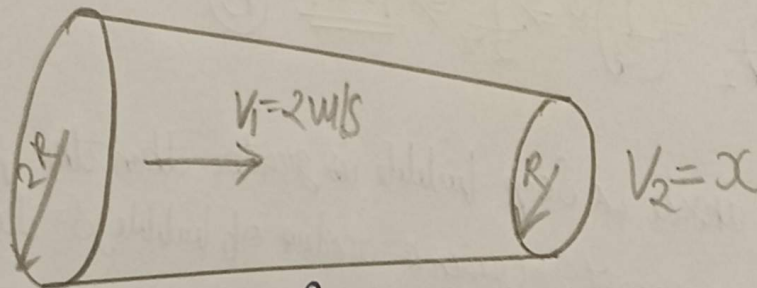
Q.1)



A tube of length L is shown in the figure. The radius of cross-section at point ① is 2 cm and at point ② is 1 cm , respectively.

If the velocity of water entering at point ① is 2 m/s , then velocity of water leaving point ② will be:- [JEE Main 2025]

Ans: (*)



$$\Rightarrow A_1 V_1 = A_2 V_2 \text{ \{Formula\}}$$

$$\Rightarrow 2\pi(2R)^2 \cdot 2 = V_2 \pi R^2$$

$$\Rightarrow V_2 = 8 = \underline{\underline{8\text{ m/s}}}$$

Q.2) The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volume of first and second bubble is:- [JEE Main 2024]

(A) 1:9

(B) 1:27

(C) 1:81

(D) 1:3

Ans: (*) $P = \frac{4T}{R}$ {excess pressure inside soap bubble?}

(2)

$T \rightarrow$ Surface Tension

(*) It is having inner and outer surfaces each contributing $\frac{2T}{r}$ to the pressure

$$P_1 = 3P_2 \text{ {Given}}$$

$$\frac{4T}{r_1} = 3 \times \frac{4T}{r_2}$$

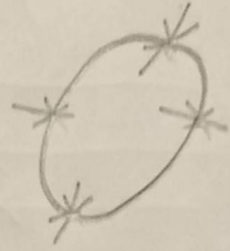
$$\frac{1}{r_1} = \frac{3}{r_2}$$

$$\frac{r_2}{r_1} = 3$$

$$V = \frac{4}{3}\pi r^3 \text{ {Volume of sphere = Volume of soap bubble}}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{1}{3}\right)^3 \Rightarrow \frac{1}{27} \Rightarrow \underline{\underline{1:27}} \text{ (B)}$$



Q.3) Pressure inside a soap bubble is greater than the pressure outside by an amount :- (given R = radius of bubble, S = Surface tension of bubble). [JEE Main 2024]

Ans: (*) $\Delta P = \frac{4S}{R}$ {Difference in pressure relating to surface tension formula?}

(*) '4' comes from inner and outer surface, for each surface pressure is given by $\frac{2S}{R}$. Thus for two surfaces :-

$$\Rightarrow 2\left(\frac{2S}{R}\right) \Rightarrow \boxed{\frac{4S}{R}}$$

Mechanical Properties of fluids

1. A hydraulic press can lift 100kg when a mass 'm' is placed on the smaller piston. It can lift _____ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

$\Rightarrow A_1 = \text{Area of smaller piston}$

Pascal's Law

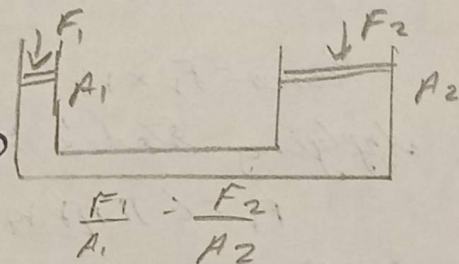
$$\frac{mg}{A_1} = \frac{100g}{A_2}$$

$$\frac{4mg}{\pi d_1^2} = \frac{1(100g)}{\pi d_2^2} \Rightarrow mg = \frac{100g}{4} \quad \text{--- (1)}$$

$d_1 = \text{diameter of smaller piston}$

$d_2 = \text{diameter of larger piston}$

According to question, $d_2' = 4d_2$, $d_1' = \frac{d_1}{4}$



$\therefore \text{New case, } \frac{4mg}{\pi d_1'^2} = \frac{4F'}{\pi d_2'^2}$

$$\frac{4mg}{\pi (d_1/4)^2} = \frac{4F'}{\pi (4d_2)^2}$$

$$\Rightarrow \frac{16mg}{\pi d_1^2} = \frac{F'}{\pi 16d_2^2} \quad \text{--- (2)}$$

Evaluating (1) & (2),

$$16 \left(\frac{100g}{\pi d_2^2} \right) = \frac{F'}{16(\pi d_2^2)}$$

$$\therefore F' = 16 \times 16 \times 1000$$

$$\therefore \text{Mass}_{\text{new}} = \boxed{25600 \text{ kg}}$$

2. A small spherical ball of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. What is the rate of production of heat when the ball attains terminal velocity?

$$\Rightarrow \text{Terminal Velocity } (v_t) = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$v_t \propto r^2$$

Rate of heat production,

$$\text{Power} = F_v \times v_t \quad (F_v \text{ is viscous force})$$

Applying Stoke's Law,

$$F_v = 6\pi\eta r v_t$$

Hence,

$$F_v \propto r v_t, \quad v_t \propto r^2,$$

$$F_v \propto r \cdot r^2 = r^3$$

$$\therefore \text{Power} \propto r^3 \cdot r^2 = \underline{r^5}$$

$\therefore$ The rate of heat production is proportional to r^5 .

3. A capillary tube of radius r is immersed in water and water rises to a height h . The mass of water in the capillary is M . If the radius of the tube is doubled, what will be the new mass of water in the tube?

$$\Rightarrow \text{Capillary Rise Formula (h)} = \frac{2T \cos \theta}{r \rho g}$$

$$\Rightarrow h \propto \frac{1}{r} \quad (1)$$

$$\text{Mass of water (M)} = \text{Volume} \times \text{Density}$$
$$\therefore M = (\pi r^2 h) \rho \quad (2)$$

Substituting (1) in (2), we get,

$$M \propto r^2 \times \frac{1}{r} \Rightarrow M \propto r$$

Since the mass M is directly proportional to the radius r , if the radius is doubled ($2r$), the mass will also double ($2M$).

$\therefore$ The new mass = $2M$

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MECHANICAL PROPERTIES OF FLUIDS

Q.1. A solid sphere of radius R acquires a terminal velocity v_1 when falling through a viscous fluid having a coefficient of viscosity η . The sphere is broken into 27 identical pieces. If each of these acquires a terminal velocity v_2 when falling through the same fluid, ratio $(\frac{v_1}{v_2}) = ?$

Ans. $27 \times \left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3}\right)(\pi R^3)$

$$\Rightarrow r = R/3$$

$$\therefore v \propto r^2$$

$$\therefore \frac{v_1}{v_2} = \frac{R^2}{r^2} = \left[\frac{R}{R/3}\right]^2 = 9$$

$$\Rightarrow \underline{\underline{\frac{v_1}{v_2} = 9}}$$

Q.2. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If radius of vessel is 5 cm & its rotational speed = 2 rotations/s, difference between centre & sides, in cm, will be $\rightarrow$

Ans. $2gh = \omega^2 r^2$

$$h = \frac{\omega^2 r^2}{2g}$$

$$\omega = 2\pi f$$

$$\therefore h = \frac{R \times \pi^2 (5 \times 10^{-2})^2}{20} = \underline{\underline{2 \text{ cm}}}$$

Q.3. Water is flowing continuously from a tap having an internal diameter $8 \times 10^{-3} \text{ m}$. The water velocity as it leaves the tap is $\frac{4}{10} \text{ ms}^{-1}$. The diameter of the water stream at a distance $\frac{2}{10} \text{ m}$ below tap is:

Ans. $d_1 = 8 \times 10^{-3} \text{ m}$; $v_1 = \frac{4}{10} \text{ m/s}$; $h = \frac{2}{10} \text{ m}$

$$\therefore v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{0.16 + 4} \approx \underline{2 \text{ m/s}}$$

acc. to eq. of continuity $\Rightarrow a_1 v_1 = a_2 v_2$

$$\left(\frac{\pi d_1^2}{4} \right) v_1 = \left(\frac{\pi d_2^2}{4} \right) v_2$$

$$\therefore d_2^2 = \left(\frac{v_1}{v_2} \right) (d_1)^2$$

$$\therefore d_2 = \left(\frac{\sqrt{0.16}}{2} \right) \times 8 \times 10^{-3} \text{ m} \\ \approx \underline{3.6 \times 10^{-3} \text{ m}}$$

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Mechanical Properties Of Fluids

Q) Figure shows liquid of given density flowing steadily in horizontal tube of varying cross-section. Cross-sectional areas at A and B is 1.5 cm^2 & 0.25 cm^2 if speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is: (Given, P_A & $P_B \rightarrow$ liquid pressure at points A & B respectively, $\rho = 1000 \text{ kg/m}^3$)

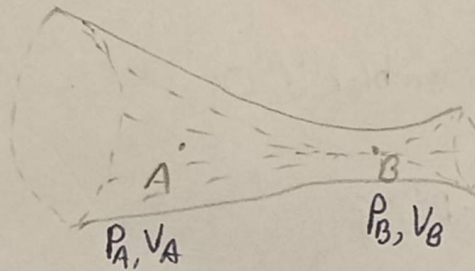
(A) 27 Pa

(B) 175 Pa

(C) 135 Pa

(D) 36 Pa

Sol.



Continuity Theorem, $A_1 V_1 = A_2 V_2$

$$1.5 \times V_1 = 25 \times 10^{-2} \times 60$$

$$V_1 = 10 \text{ cm/s}$$

Bernoulli's Theorem,

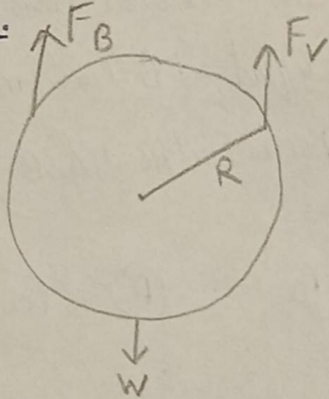
$$P_A + \frac{1}{2} \times 1000 \times (10)^2 = P_B + \frac{1}{2} \times 1000 \times (60)^2$$

$$\boxed{P_A - P_B = 175 \text{ Pa}}$$

(B) 175 Pa

Q) A spherical ball of radius r mm & density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc . Viscous force on ball when it attains constant velocity is $3696 \times 10^{-x} \text{ N}$. The value of x is:
 ($g = 9.8 \text{ m/s}^2$ & $\pi = \frac{22}{7}$)

Sol.



At terminal velocity, net force on ball is 0.

$$F_v = W - F_B$$

$$F_v = \frac{4}{3} \pi R^3 \rho_b g - \frac{4}{3} \pi R^3 \rho_l g$$

$$= \frac{4}{3} \pi R^3 g (\rho_b - \rho_l)$$

$$= \frac{4}{3} \times \frac{22}{7} \times (10^{-3})^3 \times 9.8 (10.5 \times 10^3 - 1.5 \times 10^3)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 10^{-9} \times 9.8 \times 9 \times 10^3$$

$$F_v = 3696 \times 10^{-7} \text{ N}$$

$$F_v = 3696 \times 10^{-x}$$

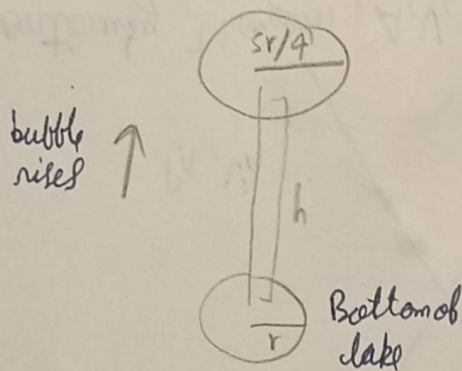
$$3696 \times 10^{-7} = 3696 \times 10^{-x}$$

$$\boxed{x=7}$$

Q) When air bubble of radius 'r' rises from bottom to surface of lake its radius becomes $\frac{5r}{4}$. Taking atmospheric pressure equal to 10m height of water column, the depth of lake would approximately be: (ignore surface tension and effect of temperature)

- (A) 11.2m (B) 8.7m (C) 9.5m (D) 10.5m

Sol.



Neglecting effect of temp.

$$10\cancel{sg} + \cancel{sg}h = 10\cancel{sg} \times \frac{125}{64}$$

$$10 + h = \frac{125}{64} \times 10$$

$$h = \frac{1250 - 100}{64} = 9.5312$$

$$\therefore h \approx 9.5m$$

$$\text{(C) } 9.5m$$

Pressure in bubble at top = P_2

$$P_2 = P_0 + \frac{4T}{5r/4}, V_2 = \frac{4\pi}{3} \left(\frac{5r}{4}\right)^3$$

Pressure in bubble at bottom = P_1

$$P_1 = P_0 + \frac{4T}{r} + \cancel{sg}h$$

$$P_1 V_1 = P_2 V_2$$

$$\& V_1 = \frac{4\pi}{3} r^3$$

$$\rightarrow \left(P_0 + \frac{4T}{r} + \cancel{sg}h\right) \frac{4\pi r^3}{3} = \left(P_0 + \frac{4T}{5r/4}\right) \frac{4\pi \times 125r^3}{64}$$

$$\& P_0 = 10\cancel{sg}$$

$$10\cancel{sg} + \frac{4T}{r} + \cancel{sg}h = \left(10\cancel{sg} + \frac{16T}{5}\right) \frac{125}{64}$$

Properties of fluids

① A cubical block of density $\rho_b = 600 \text{ kg/m}^3$ floats in a liquid of density $\rho_l = 900 \text{ kg/m}^3$. If the height of block is $H = 8 \text{ cm}$ then height of the submerged part is _____ cm.

Ans:-

$$W_b = mg ; F_B = \rho_l V_s g$$

$$= \rho_b \times V_{\text{total}} \times g$$

∴ For Flotation:-

$$W = F_B$$

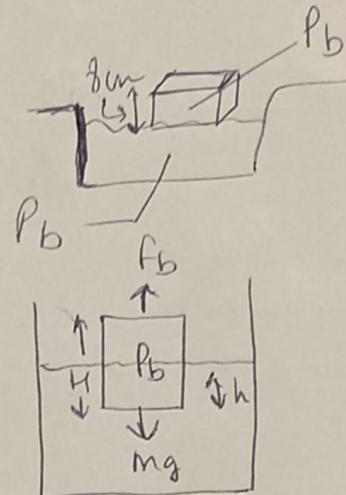
$$\Rightarrow \rho_b V_{\text{total}} g = \rho_l V_s g$$

$$\Rightarrow \rho_b V_{\text{total}} = \rho_l V_{\text{submerged}}$$

$$\Rightarrow \rho_b \times H = \rho_l \times h$$

$$\Rightarrow h = \frac{\rho_b}{\rho_l} \times H = \frac{600}{900} \times 8 = 5.33 \text{ cm}$$

∴ Height of submerged part is approx. 5.33 cm



② A brass wire of length 2m and radius 1mm at 27°C is held taut between two rigid supports. Initially it was cooled to a temp. of -43°C creating a tension T in the wire. The temp. to which the wire has to be cooled in order to increase the tension in it to $1.4T$, is _____ $^\circ\text{C}$.

Ans:-

$$\Delta L = L \alpha \Delta T \quad [\alpha \rightarrow \text{coefficient of Linear Expansion}]$$

$$\text{Strain } (\epsilon) = \frac{\Delta L}{L} = \alpha \Delta T$$

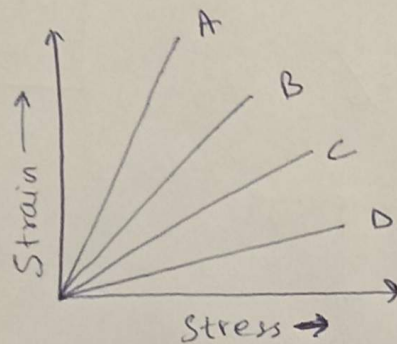
$$\therefore Y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{stress } (\sigma) = Y \times \text{strain} = Y \alpha \Delta T$$

$$\Rightarrow F/A = Y \alpha \Delta T$$

$$\Rightarrow F = Y \alpha \Delta T A \Rightarrow F \propto \Delta T \Rightarrow \frac{F_1}{F_2} = \frac{\Delta T_1}{\Delta T_2} = \frac{27 + 43}{27 - t_2}$$

$$\Rightarrow \frac{T}{1.4T} = \frac{70}{27 - t_2} \Rightarrow t_2 = -71^\circ \text{C}$$

③ The stress-strain plot for materials A, B, C, D is shown in the figure. Which material has the largest Young's modulus?



Ans:- C has the largest Young's modulus.

$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{\text{stress}}{\text{strain}}$$

$$\Rightarrow \text{Slope} = \frac{1}{Y}$$

$\therefore$ Slope is inversely proportional to Young's modulus.

$\therefore$ Material A has smallest Y due to high slope.

Material C has largest Y due to largest slope.

MECHANICAL PROPERTIES OF FLUIDS

(1) A liquid of density ρ is filled in a U-tube. One arm has a cross-sectional area A , and the other arm has a cross-sectional area $2A$. A small mass m is placed on the liquid in narrower of the arm. The difference in liquid levels in the two arms is

- (A) $\frac{m}{\rho A}$ (B) $\frac{m}{2\rho A}$ (C) $\frac{m}{3\rho A}$ (D) $\frac{2m}{3\rho A}$

Answer: Pressure due to mass m on narrow arm $= \frac{mg}{A}$

• The pressure causes height difference h in liquid columns:

$$\rho gh = \frac{mg}{A}$$

$$h = \frac{m}{\rho A}$$

Option (A)

(2) A spherical ball of radius r is falling through a viscous medium. The terminal velocity is given by:-

- (A) $\frac{2r^2(\rho - \sigma)g}{9\eta}$ (B) $\frac{2r^2(\rho + \sigma)g}{9\eta}$ (C) $\frac{2r^3(\rho - \sigma)g}{9\eta}$ (D) $\frac{2r(\rho - \sigma)g}{9\eta}$

(where ρ = density of sphere, σ = density of fluid, η = viscosity)

Ans:- Terminal velocity formula (Stokes' law)

$$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Option (A)

3) Water flows through a horizontal pipe of non-uniform cross-section. At one point the velocity of water is 2 m/s and pressure is $4 \times 10^5\text{ Pa}$. At another point, velocity is 4 m/s . The pressure at this point is (density of water 1000 kg/m^3)

(A) $3.6 \times 10^5\text{ Pa}$ (B) $3.2 \times 10^5\text{ Pa}$ (C) $2.0 \times 10^5\text{ Pa}$ (D) $1.6 \times 10^5\text{ Pa}$

Answer :- Apply Bernoulli's theorem

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

Substituting

$$4 \times 10^5 + \frac{1}{2} (1000) (2^2) = P_2 + \frac{1}{2} (1000) (4)^2$$

$$4 \times 10^5 + 2000 = P_2 + 8000$$

$$P_2 = 402000 - 8000 = 394000\text{ Pa}$$

$$\text{Approx} = 3.94 \times 10^5\text{ Pa}$$

$$\text{closest option} = 3.6 \times 10^5\text{ Pa}$$

Option A

Mechanical Properties of Fluids

Q.1 A solid sphere of radius R acquires a terminal velocity v_1 when falling through a viscous fluid having a coefficient of viscosity η . The sphere is broken into 27 identical spheres. If each of them acquires a terminal velocity v_2 , when falling through the same fluid, the ratio (v_1/v_2) equals

$$\text{Ans: } 27 \times \frac{4}{3} \pi R^3 = \frac{4}{3} \pi R^3$$

$$\text{or } r = \frac{R}{3}$$

Terminal velocity, $v \propto r^2$

$$\therefore \left(\frac{v_1}{v_2} \right) = \left(\frac{R^2}{r^2} \right)$$

$$\frac{v_1}{v_2} = \left(\frac{R}{\frac{R}{3}} \right)^2 = 9$$

$$\frac{v_1}{v_2} = 9$$

Q.2 Water is flowing continuously from a tap having an internal diameter $8 \times 10^{-3} \text{ m}$. The water velocity as it leaves the tap 0.4 ms^{-1} . The diameter of the water stream at a distance $2 \times 10^{-1} \text{ m}$ below the tap is close to.

Ans: $d_1 = 8 \times 10^{-3} \text{ m}$

$v_1 = 0.4 \text{ ms}^{-1}$, $h = 0.2 \text{ m}$

Acc to equation of motion, $v_2 = \sqrt{v_1^2 + 2gh}$
 $= \sqrt{(0.4)^2 + 2 \times 10 \times 0.2} = 2 \text{ ms}^{-1}$

Acc to equation of continuity, $a_1 v_1 = a_2 v_2$

$\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$
 $= \left(\sqrt{\frac{0.4}{2}} \right) \times 8 \times 10^{-3} \text{ m}$
 $= 3.6 \times 10^{-3} \text{ m}$

Q.3 A submarine experiences a pressure of $5.05 \times 10^6 \text{ Pa}$ at depth of d_1 in a sea. When it goes further to a depth of d_2 , it experiences a pressure of $8.08 \times 10^6 \text{ Pa}$, then $d_1 - d_2$ is approx.

$$p_1 = p_0 + \rho g d_1$$

$$p_2 = p_0 + \rho g d_2$$

$$\Delta p = p_2 - p_1 = \rho g \Delta d$$

$$(8.08 \times 10^6 - 5.05 \times 10^6) = 10^3 \times 10 \times \Delta d$$

$$3.03 \times 10^6 = 10^3 \times 10 \times \Delta d$$

$$\Delta d = 303 \text{ m}$$

Properties of Fluid

Q1. Water flows through a horizontal pipe. At one point, speed is 2 m/s and pressure is 2000 Pa.

At another point, speed is 4 m/s. Find the pressure at the second point. (Given density of water = 1000 kg/m³)

Solution:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$2000 + \frac{1}{2} (1000)(2^2) = P_2 + \frac{1}{2} (1000)(4^2)$$

$$2000 + 2000 = P_2 + 8000$$

$$4000 = P_2 + 8000$$

$$\boxed{P_2 = -4000 \text{ Pa}}$$

Q2. A small sphere of radius 1 mm falls through a viscous liquid and attains a terminal velocity of 2 m/s. If another sphere of the same material but radius 2 mm is dropped in the same liquid, find its terminal velocity.

Sol:

$$v \propto r^2$$

$$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2}{1}\right)^2 = 4$$

$$v_2 = 4 \times v_1 = 4 \times 2 = 8 \text{ m/s}$$

Terminal velocity = 8 m/s

Q. Water flows through a pipe. At point A, the speed is 3 m/s and pressure is $1.5 \times 10^5 \text{ Pa}$. At point B, which is 5 m higher than A, the speed is 5 m/s . Find pressure at point B.

($\rho = 1000 \text{ kg/m}^3$, $g = 10 \text{ m/s}^2$).

Sol:

$$P_A + \frac{1}{2} \rho v_A^2 + \rho g h_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g h_B.$$

take $h_A = 0$, $h_B = 5$:

$$1.5 \times 10^5 + \frac{1}{2} (1000) (3)^2 = P_B + \frac{1}{2} (1000) (5)^2 + (1000) (10 \times 5).$$

$$1.5 \times 10^5 + 4500 = P_B + 12500 + 50000.$$

$$154500 = P_B + 62500$$

$$P_B = 92000 \text{ Pa}$$

- Q. 2 water drops each of radius 'R' coalesce to form a bigger drop. If 'T' is the surface tension, the surface tension, the surface energy released in this process is:
Sol: When 2 droplets merge to form a larger droplet, the total vol. is conserved, therefore:

$$2 \times \frac{4}{3} \pi R^3 = \frac{4}{3} \pi r^3$$

$$r = 2^{1/3} R$$

Initial Surface energy : $V_i = 2 \times 4\pi R^2 T$

Final Surface energy : $V_f = 4\pi r^2 T = 4\pi R^2 T (2^{2/3})$

Energy released : $V_i - V_f = 4\pi R^2 T [2 - 2^{2/3}]$

- Q. A 400g solid cube having an edge of length 10cm floats in water. How much volume of the cube is outside the water?

Sol: $Mg = F_B \Rightarrow (400 \times 10^{-3}) = 10^3 \times V_d$

V. of cube under water : (V_d)

$$V_d = 400 \times 10^{-6} \text{ m}^3$$

$$\text{Total Vol: } (10 \times 10^{-2})^3$$

Subtract vol. under the water from the total Vol. to get vol. outside water.

$$(Vol)_{out} = (10 \times 10^{-2})^3 - 400 \times 10^{-6}$$

$$= 600 \times 10^{-6} \text{ m}^3$$

$$= \underline{\underline{600 \text{ cm}^3}}$$

Q. Water flows in a horizontal ~~pipe~~ pipe whose end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1 . The reading of the pressure ~~is~~ gauge ~~attached to the pipe is~~ P_1 falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to:

Sol: When water flows from a region of higher pressure to a region of lower pressure, the diff in pressure is converted into K.E of the flowing water. Acc. to Bernoulli's principle for horizontal flow this conversion can be expressed as:

$$\frac{1}{2} \rho v^2 = P_1 - P_2$$

By solving for v : $v = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$

$$v \propto \sqrt{P_1 - P_2}$$

$$\therefore \underline{\underline{\sqrt{P_1 - P_2}}}$$

Q.) Eight mercury drops, each of radius ' r ' coalesce to form bigger drop. The surface energy released in this process is _____

(A) $8\pi r^2 S$

(C) $16\pi r^2 S$

(B) $64\pi r^2 S$

(D) $4\pi r^2 S$

$$8 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$R^3 = 8r^3$$

$$R = 2r$$

$$U_i = 32\pi r^2 S$$

$$U_f = 16\pi r^2 S$$

$$\text{Ans} \rightarrow 16\pi r^2 S$$

Q.) The surface tension of soap solution is $3.5 \times 10^{-2} \text{ N/m}$. The work required to increase the radius of a soap bubble from 1 cm to 2 cm is $\alpha \times 10^{-6} \text{ J}$. The value of α is _____ ($\pi = 22/7$)

Ans $\rightarrow$

$$W = T \Delta A$$

$$\Delta A = 2 \times 4\pi (r_2^2 - r_1^2)$$

$$W = 3.5 \times 10^{-2} \times 8\pi (24 \times 10^{-4} - 1 \times 10^{-4})$$

$$W = 84\pi \times 10^{-6}$$

$$W = 264 \times 10^{-6} \text{ J}$$

$$\alpha = 264$$

Q.) A liquid drop of diameter 2mm breaks into 512 droplets. The change in surface Energy is $\alpha \times 10^{-6} \text{ J}$. The value of α is _____

$$\text{Ans} \rightarrow D = 2 \text{ mm}, R = 10^{-3} \text{ m}$$

$$R = 8r = r = \frac{R}{8}$$

$$A_i = 4\pi R^2$$

$$A_f = 32\pi R^2$$

$$\Delta A = 28\pi R^2$$

$$\Delta u = T \Delta A$$

$$= 7.04 \times 10^{-6}$$

$$\alpha = 7$$

Mechanical Property of Fluid.

SAANVI

Water flows steadily through a horizontal pipe of varying cross-section. At point A, the radius of pipe is r , and the speed of water is v . At point B, the radius is $2r$.

The pressure diff. b/w A and B is found to be

$$\Delta P = P_B - P_A$$

Find ΔP in terms of ρ (density of water) and v

$$\Rightarrow A_1 v_1 = A_2 v_2$$

$$A_1 = \pi r^2, v_1 = v, A_2 = \pi (2r)^2 = 4\pi r^2$$

$$\pi r^2 \cdot v = 4\pi r^2 \cdot v_2$$

$$v_2 = \frac{\pi r^2 \cdot v}{4\pi r^2} \quad \left| \quad v_2 = \frac{v}{4} \right|$$

$$\Rightarrow P_A + \frac{1}{2} \rho v_1^2 = P_B + \frac{1}{2} \rho v_2^2$$

$$= P_A + \frac{1}{2} \rho v^2 = P_B + \frac{1}{32} \rho v^2$$

$$P_B - P_A = \frac{1}{2} \rho v^2 - \frac{1}{32} \rho v^2$$

$$= \left(\frac{16-1}{32} \right) \rho v^2$$

$$\left| \Delta P = 15 \rho v^2 \right|$$

Q Two Spherical balls A and B of the same mass but diff. radii r and $2r$ respectively fall through a viscous liquid. If terminal velocity of ball A is v , find terminal velocity of ball B.

$$\Rightarrow v_t = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$$

$$m = \frac{4}{3} \pi r^3 \rho \text{ is constant } \rho \propto \frac{1}{r^3}$$

$$\rho_B = \frac{\rho_A}{8}$$

$$\frac{v_B}{v_A} = \frac{(2r)^2(\rho_A/8 - \sigma)}{r^2(\rho_A - \sigma)}$$

$$= \frac{4 \cdot \rho_A/8}{\rho_A} = \frac{1}{2}$$

$$\boxed{v_B = \frac{v_A}{2}}$$

Q Water flows through a horizontal pipe. At a narrow section, velocity is 10 m/s and pressure is $2 \times 10^5 \text{ Pa}$. At a wider section velocity becomes 2 m/s . Find the pressure at wider section. (Density of ~~water~~ ^{water} = 1000 kg/m^3)

$$\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$2 \times 10^5 + \frac{1}{2} (1000) (10)^2 = P_2 + \frac{1}{2} (1000) (2)^2$$

$$(2 \times 10^5 + 50000) - 2000 = P_2$$

$$\boxed{P_2 = 2.48 \times 10^5 \text{ Pa}}$$

Q1) Find the work done in blowing a soap bubble of surface tension 0.06 N/m^{-1} from 2 cm radius to 5 cm radius.

A) Given, $S = 0.06 \text{ N/m}$, $r_1 = 2 \text{ cm} = 0.02 \text{ m}$, $r_2 = 5 \text{ cm} = 0.05 \text{ m}$

$$A_1 = 2 \times 4\pi r_1^2 = 2 \times 4\pi (0.02)^2 = 32\pi \times 10^{-4} \text{ m}^2$$

$$A_2 = 2 \times 4\pi r_2^2 = 2 \times 4\pi (0.05)^2 = 200\pi \times 10^{-4} \text{ m}^2$$

$$\Delta A = A_2 - A_1 = 200\pi \times 10^{-4} - 32\pi \times 10^{-4} = 168\pi \times 10^{-4} \text{ m}^2$$

$$W = \text{Surface tension} \times \text{Increase in surface area}$$

$$= 0.06 \times 168\pi \times 10^{-4} = 0.003168 \text{ J}$$

$$\therefore \text{Work done} = 3.168 \times 10^{-3} \text{ J}$$

Q2) At what depth in an ocean, will a tube of air having one-fourth volume it will have on reaching the surface?
($1 \text{ atm} = 76 \text{ cm of Hg}$, $D_{\text{Hg}} = 13.6 \text{ g/cc}$)

A) $P_1 = 76 \text{ cm of Hg}$ (at surface)

$P_2 = (76 + \frac{h}{13.6}) \text{ cm of Hg}$ (at depth h)

Boyle's law,

$$P_1 V_1 = P_2 V_2$$

$$76V = \left(76 + \frac{h}{13.6}\right) \frac{V}{4}$$

$$306 - 76 = \frac{h}{13.6}$$

$$h = 228 \times 13.6 = 3100.8 \text{ cm},$$

At 3100.8 cm , it becomes $\frac{1}{4}$ th volume.

Q3) Two capillary tubes of length 15 cm and 5 cm and radii 0.06 cm and 0.02 cm respectively are connected in series. If the pressure difference across the end faces is equal to the pressure of 15 cm high water column, then find pressure difference across the 1) first and 2) second tube.

A) Poiseuille's formula for flow of liquid to radius r gives:

$$V = \frac{\pi P r^4}{8 \eta l}$$

$$r_1 = 0.06 \text{ cm}$$

$$r_2 = 0.02 \text{ cm}$$

$$l_1 = 15 \text{ cm}$$

$$l_2 = 5 \text{ cm}$$

$$V_1 = \frac{\pi P_1 r_1^4}{8 \eta l_1}$$

$$V_2 = \frac{\pi P_2 r_2^4}{8 \eta l_2}$$

$$V_1 = V_2 \quad (\because \text{connected in series})$$

$$\frac{\pi P_1 r_1^4}{8 \eta l_1} = \frac{\pi P_2 r_2^4}{8 \eta l_2} \quad [P_1 = (15-h) \rho g, P_2 = h \rho g]$$

$$\frac{\pi (15-h) \rho g \times r_1^4}{8 \eta l_1} = \frac{\pi \cancel{15} h \rho g \times r_2^4}{8 \eta l_2}$$

$$\frac{(15-h) r_1^4}{l_1} = \frac{h r_2^4}{l_2}$$

$$\frac{15-h \times (0.06)^4}{153} = \frac{h (0.02)^4}{5}$$

$$h = (15-h) 27$$

$$h = \frac{405}{28} = 14.464 \text{ cm}$$

$$1) \Delta P_1 = 15 - 14.464 = 0.536 \text{ cm of water column} //$$

$$2) \Delta P_2 = \cancel{15} 14.464 \text{ cm of water column} //$$